Five-Year Integrated M. Sc. Examination-2024

Subject: Mathematics

Paper: MT-3-5-5 (2016)

(Mathematical Theory of Probability & Statistics)

Time: Three Hours **Full Marks: 40**

Questions are of values as indicated in the margin.
Notations and symbols have their usual meanings.

Answer ***any four*** questions.

1. (a) Give the axiomatic definition of probability and use axioms to deduce the classical definition of probability. [3+3]

   (b) For any two events A_1 and A_2 connected to a random experiment, show that $P(A_1 + A_2) = P(A_1) + P(A_2) - P(A_1 A_2)$. [2]

   (c) A die is rolled once. If the result is either an even face or a multiple of three, I win. What is the probability of my winning? [2]

2. (a) Define conditional probability. Hence deduce the multiplication rule of probability for any three events A_1, A_2, and A_3. When they are said to be mutually independent? [2+3+3]

   (b) For any event A, if $P(A) = 0$, does it imply that A is an impossible event? Justify your answer. [2]

3. (a) State and prove Bayes' theorem. [2+3]

   (b) There are three identical urns containing white and black balls. The first urn contains 2 white and 3 black balls, the second urn 3 white and 5 black balls, and the third urn 5 white and 2 black balls. An urn is chosen at random, and a ball is drawn from it. If the ball drawn is white, what is the probability that the second urn is chosen? [5]

4. (a) Define distribution function, $F(x)$ of a random variable X. Show that for any two real numbers a and b, $P(a \le X \le b) = F(b) - F(a-0)$. [2+3]

   (b) Show that a function $f(x)$, given by,

$$f(x) = \begin{cases} x & 0 < x < 1, \\ k - x & 1 < x < 2, \\ 0 & \text{elsewhere}, \end{cases}$$

   is a probability density function for a suitable value of the constant k. Calculate the probability that the random variable lies between 1/2 and 3/2. [2+3]

5. (a) Define mean and variance of a random variable and give their physical significances. [3+3]

   (b) Calculate the mean and variance of a normal (m, σ) variate. [2+2]

6. (a) Define moment generating function (mgf) of a random variable X. A continuous distribution has probability density $f(x) = ae^{-ax}$ $(0 < x < \infty,\ a > 0)$. Calculate the mgf, and hence obtain the k-th order moment (α_k) of X about the origin. [1+(1+4)]

   (b) Use the method of maximum likelihood to estimate the parameter p of a binomial (N, p) population. [4]